Prediction and validation of short- to long-term earthquake probabilities in inland Japan using the hierarchical space-time ETAS and space-time Poisson process models

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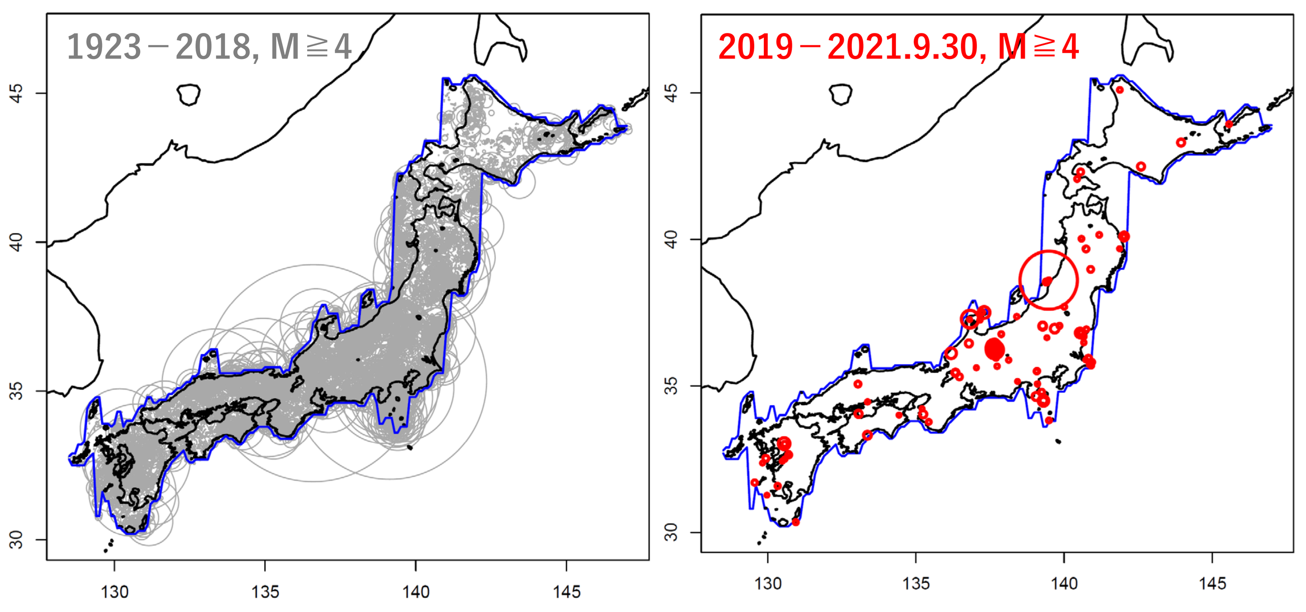
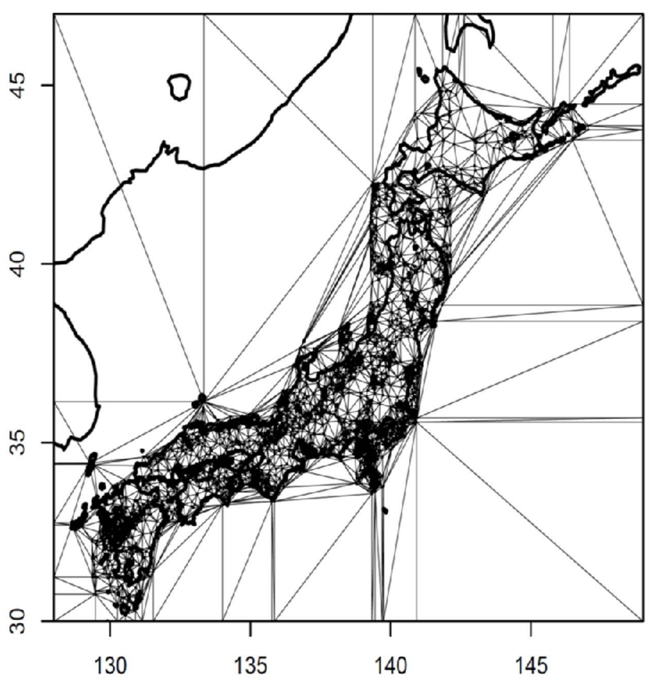




I will discuss the prediction of earthquakes directly inland.
-- The Bayesian models such as intensity functions for spatial point configurations discussed in this presentation are defined by piecewise linear functions defined on Delaunay triangular networks made from earthquake configurations, which are linearly interpolated. In this regard.
-- We have recently published the software and manual (see Ogata et al. 2021 in the abstract).

Hierarchical space-time ETAS (HIST-ETAS) model

BG rate — Aftershock Productivity

$$\lambda(t,x,y\mid H_t)=\mu(x,y)+\sum_{\{j:t_j<t\}}\frac{K_0(\bar{x}_j,\bar{y}_j)}{(t-t_j+c)^{p(\bar{x}_j,\bar{y}_j)}}\times\left\{\frac{(x-\bar{x}_j,\ y-\bar{y}_j)\ \bar{S}_j\begin{pmatrix}x-\bar{x}_j\\ y-\bar{y}_j\end{pmatrix}}{e^{\alpha(\bar{x}_j,\bar{y}_j)M_j}}+d\right\}^{-q(\bar{x}_j,\bar{y}_j)}$$

Intensity rate : Predicted number of **M≥4 earthquakes/day/square degree**, where *Ht* is the historical information of occurrences before time *t*

HIST-ETAS-5pa model : if α, p, and q depend on the position coordinates in the above equation (the weights of the constraints are 5-dimensional vectors)

HIST-ETAS-μk model : if α, p, and q are constants in the above equation (the weights of the constraints are 2-dim. vectors)

Time-independent Poisson process models:

$$\lambda(t,x,y\mid H_t)=\lambda(x,y),\quad 0\le t\le T$$

(a) **Uniform Poisson model for inland area**

$$\lambda(x,y)=\begin{cases}\hat{\lambda}_{inland} & \text{if } (x,y)\in \text{inland}\\ 0 & \text{otherwise}\end{cases}$$

(b) **Nonhomogeneous Poisson models**

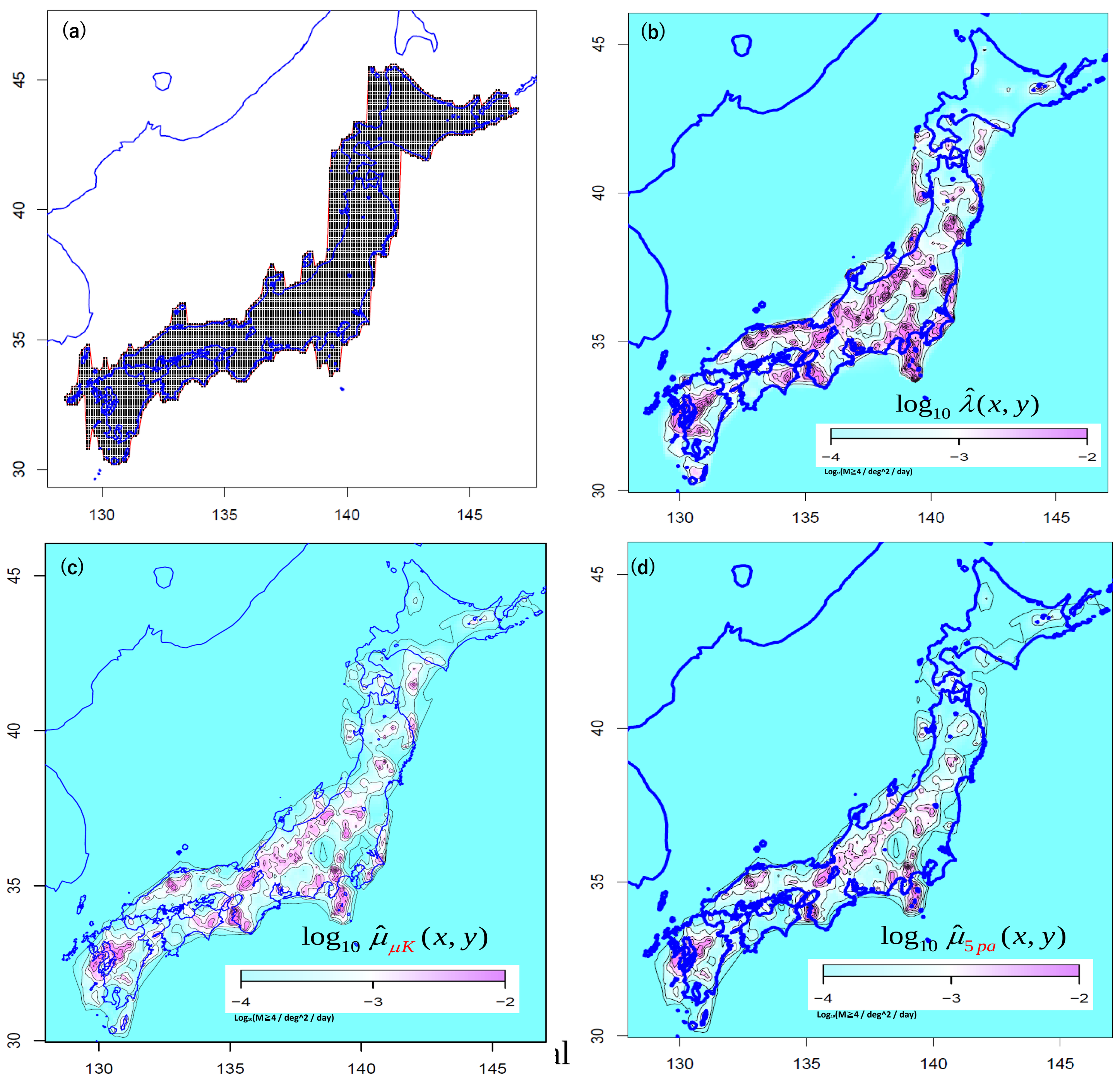
$\lambda(x,y)=\hat{\lambda}(x,y)$; optimal MAP estimate

(c) **Background rate of HIST-ETAS-μK model**

$$\lambda(x,y)\propto\hat{\mu}_{\mu K}(x,y)$$

(d) **Background rate of HIST-ETAS-5pa model**

$$\lambda(x,y)\propto\hat{\mu}_{5pa}(x,y)$$



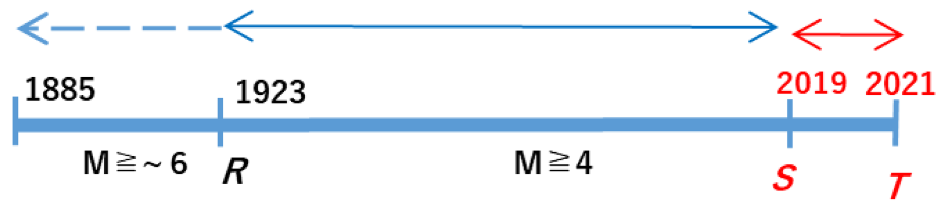
…the inland Japan, all of which are obtained by the respective MAP estimate that minimize the Akaike Bayesian Iinformation Criterion: (a) inland uniform Poisson process model, (b) non-uniform spatial Poisson process model, (c) background ¥mu intensities of the HIST-ETAS-mK model and (d) that of the HIST-ETAS-5pa model, respectively. The colors and contours are in logarithmic scale.

Short-term forecast

Log-likelihood score:

$$\ln L(S,T\mid M_i\ge M_c\ ;\ \text{Occurrence History})=\sum_{\{i;\ S<t_i<T,M_i\ge M_c\}}\log\hat{\lambda}(t_i,x_i,y_i,M_i\mid H_{t_i})-\int_{M_c}^{\infty}\int_S^T\iint_{Inland}\hat{\lambda}(t,x,y,M\mid H_t)\,dx\,dy\,dt\,dM$$

where $\hat{\lambda}(t,x,y,M\mid H_t)=\lambda_{\hat{\theta}}(t,x,y\mid H_t)10^{-\hat{b}(M_c-4.0)}$ with $b=0.9$

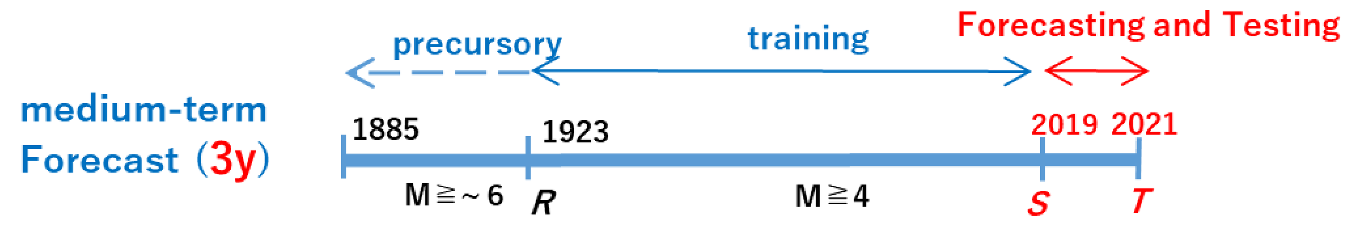


Forecast for magnitude range	M≧4.0	M≧4.5	M≧5.0	M≧5.5
Number of earthquakes	126	42	12	3
HIST-ETAS-μK	551.2	252.5	49.9	-3.2
HIST-ETAS-5pa	638.7	276.6	55.8	-3.9
Inland uniform Poisson	0.0	0.0	0.0	0.0
Non-uniform Poisson	157.9	105.6	41.7	9.3

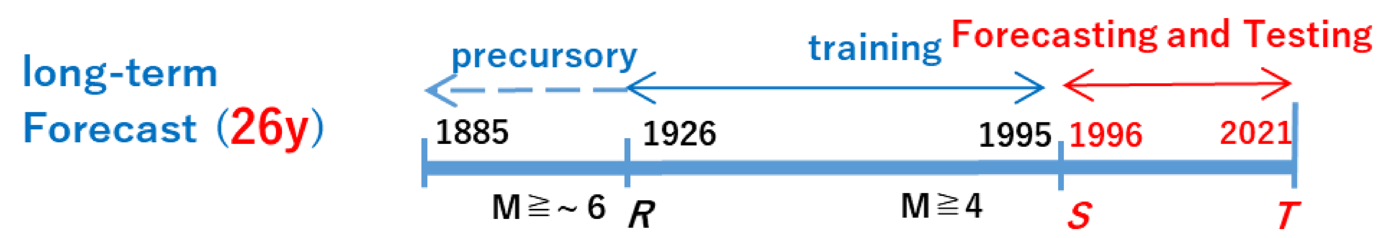
The number is the "log-likelihood score" which compares the performance relative to the score for the inland uniform Poisson process, which is set at 0. -- The larger the score, the better the score, with red being the best. -- HIST-ETAS-5pa is the best at predicting magnitudes from 4.0 to 5.0, followed by HIST-ETAS-μK.

Spatial probability density score

$$\log\prod_{\{i;\,M_i\ge M_c\}}\hat{f}(x_i,y_i,M_i)=\log\prod_{\{i;\,M_i\ge M_c\}}\left\{\frac{\hat{\lambda}(x_i,y_i,M_i)}{\int_{M_c}^{\infty}\iint_{Inland}\hat{\lambda}(x,y,M)\,dx\,dy\,dM}\right\}$$

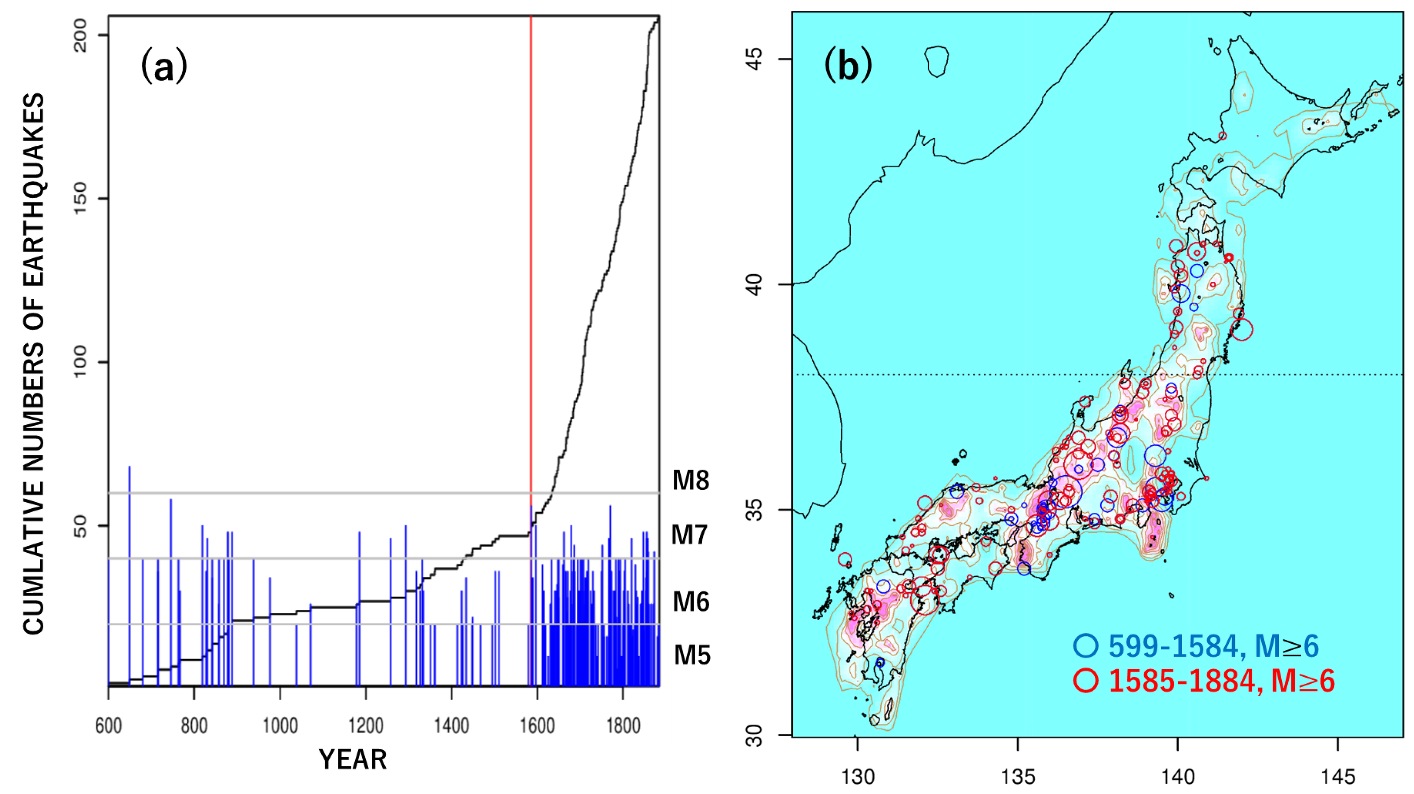


Spatial probability density I(x,y,M)	M ≥ 4.0	M ≥ 4.5	M ≥ 5.0	M ≥ 5.5
number of events	126	42	12	3
inland uniform	0.0	0.0	0.0	0.0
non-uniform	139.7	59.7	20.3	3.0
μ(x,y): HIST-ETAS-μK	91.2	41.8	9.3	-1.4
μ(x,y): HIST-ETAS-5pa	55.5	38.9	11.0	-0.9

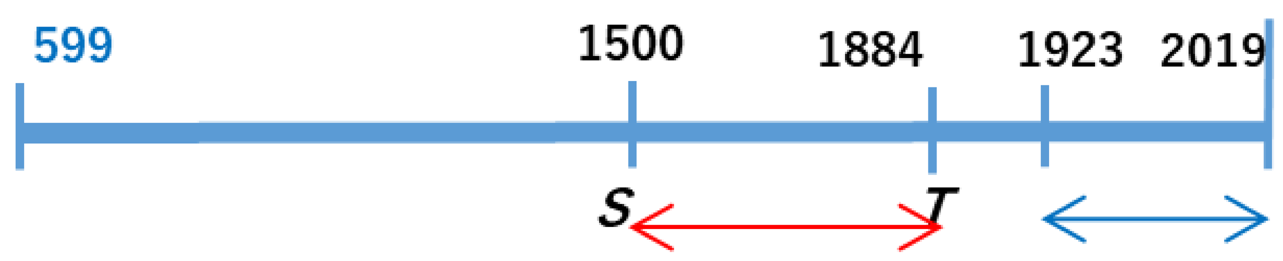


Spatial probability density I(x,y,M)	M ≥ 4.0	M ≥ 4.5	M ≥ 5.0	M ≥ 5.5	M ≥ 6.0	M ≥ 6.5	M ≥ 7.0
number of events	2765	990	305	103	43	18	5
inland uniform	0.0	0.0	0.0	0.0	0.0	0.0	0.0
non-uniform	2835.0	1024.9	264.8	60.1	12.3	9.8	2.5
μ(x,y): HIST-ETAS-μK	2270.7	843.5	230.0	53.7	16.9	9.2	3.0
μ(x,y): HIST-ETAS-5pa	2576.7	954.1	263.7	61.6	19.9	10.6	2.5

Historical record（Utsu） **599 - 1884**



Reverse forecasting & Testing   training



Forecast of magnitude range	All events	M ≥ 5.5	M ≥ 6.0	M ≥ 6.5	M ≥ 7.0	M ≥ 7.5
number of events	131	129	114	57	25	3
(a) uniform in inland n	0.0	0.0	0.0	0.0	0.0	0.0
(b) non-homogeneous	-7.8	-11.3	-15.7	-16.3	-12.4	-1.2
(c) HIST-ETAS-mK BG	32.9	28.1	24.0	9.8	4.0	1.1
(d) HIST-ETAS-5pa BG	36.7	31.8	29.4	12.5	5.5	1.6